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Формирование магнитосферных возмущений как результат системной динамики геомагнитного хвоста

Аннотация. Рассмотрено выделение энергии в геомагнитном хвосте магнитосферной суббури. В этом подходе геомагнитный хвост рассматривается как открытая термодинамическая система с бесконечным числом связей с внешней космической средой. Ранее была представлена самосогласованная общая теория выделения энергии в такой системе. В отличие от существующих моделей, в представленной модели суббури считается, что на её развитие влияют два фактора. Первый – это традиционный разворот межпланетного магнитного поля (ММП) Bz на юг, приводящий к усилению энергообмена между хвостом и окружающей средой. Второй фактор определяется свойствами энергетической эволюции в “хвосте” как открытой системе. Взаимодействие этих двух факторов ведет к различным типам структуризации магнитосферной активности, включая классическую 3-фазовую суббурю. Дается объяснение деталей системного механизма, лежащего за возможными формами геомагнитных возмущений, и производится сравнение с типичными характеристиками суббури.

Ключевые слова. Геомагнитный хвост, эволюция энергии, вероятностный механизм, термодинамическая система, фаза суббури.

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Original article

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Shaping of Magnetospheric Disturbances by System Behaviour of Geomagnetic Tail

Abstract. Development of magnetospheric substorm from standpoint of energy evolution in geomagnetic tail is considered. In this approach, geomagnetic tail is taken to be an open thermodynamic system with infinite number of conservation energy links to external space environment. Self-consistent general theory of energy evolution in such system was presented earlier. In contrast to existing models of substorm development, presented model suggests believing that development is a combined effect of two influence factors. The first one is the traditional southward turn of interplanetary magnetic field (IMF) Bz resulting in the intensification of energy exchange between the tail and externals. The second agent is determined by physics of energy evolution in abovementioned system leading to the well-known structure of substorm. Intercoupling between these agents can produce different forms of magnetospheric activity including substorms. Explanation of essentials for system mechanisms beyond the known substorm signatures and comparison with the typical substorm features is also provided.

Keywords. Geomagnetic tail, energy evolution, probabilistic mechanism, thermodynamic system, substorm phase.

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1. Introduction

1.1. Generals

As it is known, the Earth's magnetic tail acts as a reservoir for the energy that is extracted in the interaction between the solar wind and the Earth's magnetosphere. Occasionally, a portion of that energy is released through a violent process known as a magnetospheric substorm. The substorm is one of the most important magnetospheric phenomena, and it is the subject of extensive research [1,2].

Under a more electromagnetic viewpoint, the distortion of the Earth's magnetic field is affected by enormous flows of charged particles. These currents flow through space across and along lines of magnetic force. One portion of this current system flows in a sheet across the centre of the magnetotail, dividing the tail into two lobes of oppositely directed magnetic field. This current is known as the cross-tail current, and it plays a central role in substorms [3,4].

The interplanetary magnetic field (*IMF*) carried by the solar wind is another critical actor in the transfer of energy to the magnetosphere. Both the magnitude and the orientation of the *IMF* are quite variable. The Earth's magnetic field points primarily northward, so when the *IMF* contains a southward component ($-B_z$), a complicated process of magnetic reconnection occurs. Reason is that at negative B_z the magnetic lines of force in the solar wind become connected with the lines of force from the Earth [5,6].

Reconnection process was first postulated in the context of solar physics and was applied to the magnetosphere by Dungey in its model of open magnetosphere [5]. The place where this interconnection occurs is called an *X*-line because of the geometry of magnetic field lines in that region. Then, the reconnected magnetic field lines are dragged antisunward by the solar wind flow and stretch out to form the lobes of the magnetotail. The field lines connected to the solar wind are referred to as "open." In the distant magnetotail, another *X*-line forms, which severs the open-lobe field lines. The reconnected field lines now have both ends attached to the Earth and are referred to as "closed." The closed lines of magnetic flux contain a relatively dense plasma that forms the plasma sheet down the center of the magnetotail.

In the geomagnetic tail through reconnection process a magnetic energy is converted into a plasma energy.

Several different processes including magnetospheric substorms have been shown to depend on the sign of B_z, and most scientists invoke reconnection with *IMF* to explain such phenomena.

Note that generally, energy exchange at reconnection can be thought as a bidirectional transfer for different forms of energy (magnetic, electrical, heat, mechanical and so on) to and from the tail, where the tail comes as a storage for gathered energy. At some point, this gathered energy must be released and this release takes form of a substorm.

In the following sections, we will present the model of magnetospheric substorm accounting above and discuss consistence of this model with existing knowledge.

1.2. Phenomenology of substorm

It is well accepted that a substorm is composed of three main phases: the growth phase, the expansion phase, and the recovery phase (e.g., [7]). The onset for isolated substorm coincides with moment of when the *IMF* turns southward, and dayside reconnection begins [7,8].

During the growth phase, energy is being extracted from the solar wind and accumulated by the magnetotail, in parallel, the cross-tail current intensifies. This intensification causes the magnetic field in the lobes to increase, and the magnetotail becomes more stressed as the field lines become stretched into a more tail-like configuration. In parallel, the near-Earth plasma sheet at ~ 15 R_E narrows down in the north-south dimension; this process is referred to as plasma-sheet thinning, where R_E is Earth radii.

The onset of the expansion phase of a substorm, as a rule, begins when in the near-Earth magnetotail the magnetic field suddenly relaxes from the stressed tail-like configuration to a more dipolar one, as if the cross-tail current, which produces the stress, has suddenly been reduced [9]. This phenomenon

is known as a dipolarization. In contrast, in the more distant magnetotail, the plasma sheet thins at substorm onset and expands only later, during the recovery phase of the substorm altogether with an increase in the northward component of the magnetic field.

At substorm onset, the cross-tail current sheet is disrupted, plasma is energized, the plasma sheet expands, and the magnetic field relaxes to a less tail-like configuration.

Current is diverted out of the magnetotail, along the magnetic field into the ionosphere, across the ionosphere via the westward-traveling surge, and back out into the magnetotail.

So, resuming above, it is possible to conclude that the tail behaves itself as a complex system, with multi-scale spatio-temporal behaviour.

1.3. Models of magnetosphere substorm

There are a few the well-accepted models of magnetospheric substorms, among them the near-Earth neutral line model, the boundary layer model, the magnetosphere-ionosphere coupling model, and the thermal catastrophe model.

Perhaps, the most popular one is the near-Earth neutral line model [9]. This model states that a southward *IMF* produces reconnection on the dayside magnetosphere and the reconnected field lines are swept tailward by the solar wind flow and added to the magnetotail. This process increases the cross-tail current and the added lobe pressure makes the plasma sheet to thin. As a next step, the region of *X*-type reconnection appears in the near-Earth tail, as a result the cross-tail current disrupts and plasmoid with the closed magnetic field lines emerges. In parallel, the bidirectional current flows between the ionosphere and magnetosphere along the field lines forming the so-called current wedge. Earthward of the neutral line, the reconnected field lines move to Earth, which leads to an expansion of the plasma sheet, and tailward the plasma sheet rapidly thins. Along with that, the plasmoid moves down the tail. The near-Earth neutral line becomes the new distant neutral line, and the plasma sheet keeps expansion behind it.

Another model for substorm is the boundary layer model [10]. This model emphasizes the role of plasma sheet boundary layer between the plasma sheet and the lobe. In this model, existence of a velocity shear in Birkeland currents flow rules. During enhanced reconnection at the distant neutral line, the velocity shear potentially may become unstable making the currents through the boundary layer to bunch up. At this, the southward B_z signatures, the *X*-line geometry of reconnection, are just the side effects of the bunched currents. Also, the boundary layer model does not provide an explanation for near-Earth phenomena.

The magnetosphere-ionosphere coupling model [11] represents a third type of substorm model. This model stresses the effects of coupling of the magnetosphere and ionosphere via Birkeland currents. Such coupling causes a great increase in the Birkeland current density, and the enhanced current produces the auroral brightening on the substorm expansion phase. In this model, substorms can occur without a new *X*-line formed in the near-Earth plasma sheet. The explosive intensification during the expansion phase is powered by releasing the magnetic energy stored on closed field lines in the plasma sheet. The stored energy is then released by the unloading instability driven by positive feedback in the substorm current wedge.

The next model is a thermal catastrophe one. It is assumed that the resonant absorption of Alfven waves in the plasma sheet boundary layer (PSBL) provides a significant source of energy for the heating of plasma sheet particles. Since the absorbed energy is a function of the central plasma sheet temperature, the heating curve yields an equation of state for the steady state plasma sheet whose solution has the form of a mathematical catastrophe. Master equation is generalized to include the dynamic terms allow evaluation of the time scale for the catastrophe to occur [12] and apply it to development of substorm. Reference should also be made to the the models lie on the basis of self-organization phenomenon where solution is found as a compromise between contradicting factors [13] accounting thermodynamic approach [14].

2. Substantiation of presented model

While the abovementioned models have provided a valuable working model for substorm phenomenology, it is not only possible description for understanding the observations.

Below we present an alternative framework in which substorm development can be understood as well.

2.1. Model assumptions

At high level of abstraction, the above-described physical processes in magnetosphere during period of substorm activity deal with different forms of energy transformations which are mainly concentrated within the tail. It prompts that, generally, the tail or the most active part of it can be considered as an open thermodynamic system which is in a permanent connection with the more (at the southward B_z) or the less (at the northward B_z) capacious energy reservoir that is supplied by the solar wind and interplanetary space. Obviously, that these connections include a huge number of influential factors of various physical nature. Under these conditions, the tail meets criteria for an open thermodynamic system with infinite number of conserved energy links with environment (*CEL* model) which has been suggested in [15], and studied in the detail in [16].

In other words, we have the sufficient reasons to apply the general *CEL* theory to interpret the known pattern of energy transformations in the tail during magnetospheric substorm.

Critical aspect of this approach is an energy link between the tail and its externals that deals with the conserved quantity of energy and therefore obeys an appropriate energy continuity equations (*ECE*).

Following this logic, the presented approach incorporates those exchange scenarios in which change in the system state is projected onto an energy axis no matter what sort of exchange process or combination of processes (thermal, chemical, optical, mechanical or any other) it deals with.

To support this approach, we have to make sure that suitable *ECE* covers any possible act of energy exchange. The guarantee of that is an infiniteness of the set of *ECE* which is generated by joined work of all implicated factors. So, we come to an infinite number of *ECE* (energy links) given by system (2).

Big difference between the single *ECE* (1) and the system of *ECE* (2) is that (1) deals with the predefined energy transfer through one fixed energy channel only, whereas (2) processes an infinite number of energy channels, where each single energy transfer is done on the random basis.

Within above logic, though energy released at reconnection process includes a few diverse components such as hydromagnetic energy, heat, electromagnetic energy, and others, further we will unify an energy exchange process and assume that variations of the generic rate of energy exchange in the tail y effectively describes all particular energy interactions.

Highlight that requirement for medium to be conducting is not a factor for *CEL* model as it is irrelevant of presence of positively and negatively charged particles, *i.e.,* plasma. Equally, *CEL* model does not assume that described system mandatorily possesses any sort of self-organized criticality.

Further, we provide necessary mathematical background for understanding of our model.

2.2. Mathematical grounds of presented model

In the light of above, the basic mathematical form of energy link between the tail and its environment is *ECE* that in the non-relativistic approximation can be written as

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \mathbf{J} = 0 \tag{1}$$

and appropriate infinite number of energy links is given by the system

$$\begin{cases} \dfrac{\partial \varepsilon_1}{\partial t} + \nabla \cdot \mathbf{J}_1 = 0 \\ \dfrac{\partial \varepsilon_2}{\partial t} + \nabla \cdot \mathbf{J}_2 = 0 \\ \cdots \\ \dfrac{\partial \varepsilon_n}{\partial t} + \nabla \cdot \mathbf{J}_n = 0 \\ \cdots \end{cases} \tag{2}$$

where $\mathbf{J}$ is flux of energy, ε is volume density of energy, t is time, and ∇ is Nabla operator, $n \to \infty$.

Solution of system (2) for efficiency of energy transformation between the tail and the rest part of its environment is

$$\Upsilon = y - y\ln y \quad (3)$$

and solution for entropy as

$$S = ln|\, d\Upsilon \,| = \ln |\ln y| \quad (4)$$

from which

$$\frac{dS}{dy} = \frac{1}{y\,|\ln y|} \quad (5)$$

and roots $\Delta S = 0$ are

$$f_{y\,in} = W_L(f_{y\,out}{}^{-f_{y\,out}}) \quad (6)$$

where $y = J/J_0$ is the total rate of exchange through the tail interface, normalization constant $J_0 > 0$, W is a Lambert function, $f_{y\,in}$ is the probability density for flux inwards the tail while $f_{y\,out}$ is the probability density for flux outwards the tail.

So, theoretical dependence (3) for $\Upsilon(y)$ is shown in Fig. 1. In physical meaning, here $\Upsilon \sim \Delta E/M$, where ΔE is change in energy of plasma sheet while M is transfer of magnetic energy from reconnection process. Hence, Υ is an efficiency of energy conversion in the tail and, thereby, describes how big is a change ΔE caused by the transferred energy M. As we see, Υ experiences a non-monotonic variation with maximum at some y.

The remarkable feature of (3) is that actually dependence $\Upsilon(y)$ consists of three different phases shown in different colour. The two obvious phases are in the y-ranges $[0, y_{MAX}]$ and $[y_{MAX}, y_T]$. However, there is one more phase due to existence of bifurcation point y_{ONSET}. As a result, the range $[0, y_{MAX}]$ consists of two phases. One is in the range $[0, y_{ONSET}]$, the second one is in the range $[y_{ONSET}, y_{MAX}]$. Note that emergence of the point y_{ONSET} is a direct consequence of more general *CEL* theory operating with the system concepts.

Indicate that the masking role of y_{ONSET} in the dependence $\Upsilon(y)$ is not accidental. Point y_{ONSET} has a few more outstanding features that reveal in dynamics of other system parameters. It allows y_{ONSET} effectively control energy dynamics of open system in *CEL* theory. Some of the features of y_{ONSET} are described below.

In the considered context, the most prominent feature of y_{ONSET} is that at this point the system (tail) drastically changes its energy behaviour. While at $y < y_{ONSET}$, the tail is in the totally unordered chaotic regime of energy accumulation, at $y > y_{ONSET}$, an energy profile of the tail is being become more structured.

General profile of dependence (6) provides understanding for the relative ratio $f_{y\,in}/f_{y\,out}$. It is seen that originally $f_{y\,in} > f_{y\,out}$, and dominance of $f_{y\,in}$ lasts in some y-range, then ratio $f_{y\,in}/f_{y\,out}$ achieves its maximum,

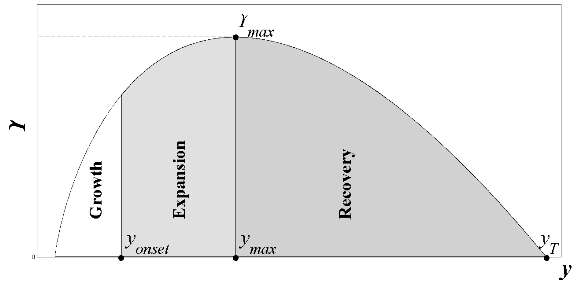

Fig. 1. Dependence of energy efficiency ϒ on general rate of energy exchange y. Three phases are shown that are associated with the observed phases of substorm of growth, expansion, and recovery. The point y_{onset} marks commencement of the expansion phase and y_{max} marks maximum of efficiency associated with the end of expansion phase. All quantities are unitless

and finally goes down. Hence, dynamics of ratio $f_{y\ in} / f_{y\ out}$ definitely demonstrates that initially $f_{y\ in}$ is higher $f_{y\ out}$, whereas later $f_{y\ out}$ starts to dominate as shown in Fig. 2 at condition $\Delta S = 0$.

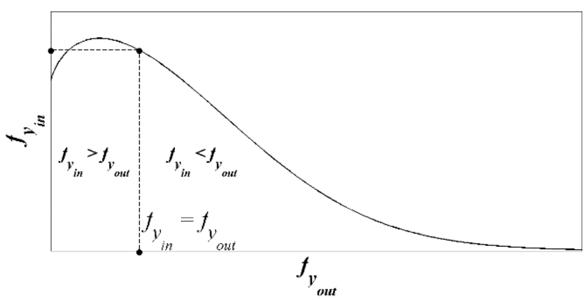

Fig. 2. Dependence of probability density for flux inwards tail $f_{y\ in}$ (vertical axis) on probability density outwards tail $f_{y\ out}$ (horizontal axis) at $\Delta S = 0$. So, at $f_{y\ in} > f_{y\ out}$ flux P_{in} is more preferable while at $f_{y\ in} < f_{y\ out}$, ratio is opposite and flux y_{out} takes priority. All quantities are unitless

Similar probabilistic trend finds confirmation in dynamics for the entropy production. Finally, as it stems from (5) an entropy production dS/dy also changes its sign at $y = y_{ONSET}$ as shown in Fig. 3. It is another argument in favour of the radical change in energy regime that occurs in the tail. Quite important fact is that such non-monotonic profile of dS/dy supports the trend in the preferable probability density for energy flux components $f_{y\ in}$ and $f_{y\ out}$ in Fig. 2

So, the point $y = y_{onset}$ comes as the severing point that indicates to the transition of the tail to the new energy regime.

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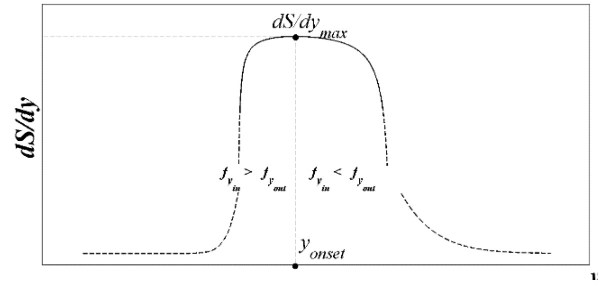

Fig. 3. Dependence of rate for entropy production dS/dy on the rate of energy exchange y. Change in the probabilistic preference from $f_{y\,in}$ to $f_{y\,out}$ takes place at $y = y_{onset}$. At this, the point y_{onset} marks commencement of expansion phase and coincides with the point for maximum entropy production dS/dy_{max}. All quantities are unitless.

To be exact, from *CEL* theory follows that the tail as a system has an infinite number of bifurcation points such as y_{max} for the $\Upsilon_{\max}$ and y_T for remote $\Upsilon = 0$, and y_{onset} is just the most prominent one among other points. However, the other bifurcation points do not have so prominent appearance, as a result, they are less likely to be recognized in observations.

Thus, passing on the point y_{onset}, but not the point y_{max}, radically changes system behaviour of the tail leading to ultimate prevalence of the processes which contribute to the loss of energy accumulated during the phase of growth. It is worth to note that despite of prevalence of the energy flux out the tail, ratio $f_{y\,in} < f_{y\,out}$ lasts until the termination of energy evolution of the tail, *i.e.,* until $y = y_T$ shown in Fig.1.

Underline that use of *CEL* theory as a formalism for energy evolution in the tail permits to explain the *3*-phase form of substorm simply following internal logic of the tail energy development. In this sense, though suggested theory needs energy to function, it is still immune to the any changes in this energy flow excepting its termination.

Another important moment is that *CEL* theory does not directly depend on the amount of energy injected into the tail, *i.e., CEL* theory does not have any threshold quantity of energy. Instead, it lies on the dynamics of internal energy flows only.

For comparison, in Fig. 4 we present a plot for time variation of *AL*-index that characterizes profile for the substorm energy evolution [17]. In this picture, we clearly see three phases of substorm. They are the growth phase P_G from ~ 10:20 UT until ~ 10:54 UT, the expansion phase P_E from ~ 10:54 UT until ~ 11:24 UT, and the recovery phase P_R from ~ 11:24 UT until ~ 13:00 UT.

Pay attention that the onset of expansion phase is not at the maximum *AL (*11:24 UT*)*. In opposite, it starts ~ 30 min before that at ~

10:54 UT, *i.e.,* in the point y_{onset}. This fact is quite aligned with the meaning of y_{onset} described above. Another observation is that at maximum of *AL* (~ 11:24 UT) the recovery phase starts. Again, it looks quite appropriate with the meaning of Υ as efficiency for energy exchange. In accordance with that, maximum of Υ is achieved when the tail has acquired all the energy sent to it from surroundings.

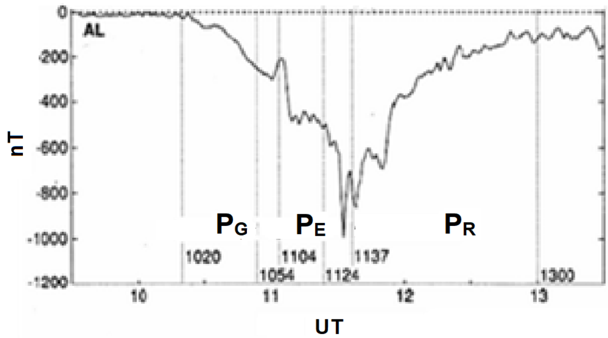

Fig. 4. Typical substorm activity in variations of AL-index obtained as a result of a superposed epoch study. The growth phase P_G starts at ~ 10:20 UT, the expansion phase P_E at ~ 10:54 UT, the recovery phase P_R at ~ 11:24 UT.

So, in presented model, condition $f_{y\,out} > f_{y\,in}$ comes as a trigger for the expansion phase which arranges energy to leave the tail. In this scenario, the reconnection process by itself can only provide energy but cannot cause a shaping of it into the well-known *3*-phase structure of substorm.

To complete the *3*-phase design, it is required to add one more element to the plain energy in – energy out cycle. This missing element can be provided by a functioning of the tails as an open system. So, the *3*-step construction of a substorm is a direct consequence of the system operation of the magnetotail.

In considered context, the condition $f_{y\,out} > f_{y\,in}$ does not depend on the sign of B_z direction and can happen at any one. However, the power of substorm and amplitude of its observable signatures are determined by conditions at reconnection area and are irrelevant of the system behavior of the tail.

So, to see a powerful substorm with the classic *3*-step form, the southward orientation of B_z and change of energy rate outwards the tail should coincide. If it is not the case, *i.e.,* at the northward orientation of B_z and preferable energy flux inwards the tail, the chance to register intense geomagnetic disturbance of classic form is a less likely.

The more patterns can occur at other combinations of above factors. If the favorable conditions for the southward reverse of B_z exist,

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but total rate y does not turn outwards the tail, then the more or less significant magnetospheric disturbance can still develop but it may lack accustomed form and *3*-phase design we used to see. Otherwise, if B_z is northwards but the rate y turns outwards then we can observe a geomagnetic reply of the "right" form but of small amplitude as observed, for example, in [16]. Pay attention that occurrence of substorms at the northward B_z was regularly reported in observations, for ex., [17, 18].

So, we see that presented model of magnetospheric substorm can describe a few geomagnetic signatures which can be observed in real experiments.

2.3. Physics of *CEL* model

It is worth noting that the presented *CEL* model deals with the high-level description of energy transformations. As such, it does not assume a capacity to dive deeply into the physical details accompanying the process of substorm development. That is why, below we focus on the analysis of the limited number of high-level physical quantities characterizing development of substorm and not contradicting directly to presented theory. As such, the high-level quantities we consider are the preferable direction of energy flows and forming of the point y_{onset}.

Accounting repeated time occurrence of substorm activity, we are obligated to assume that the conditions favourable to such activity are shaping regularly. In the context of the presented model, it means that energy exchange between the tail and surroundings in the form of multitude sources/sinks exists for a reasonable period of time. Staying within these sufficiently general assumptions, we are not limited by a physical mechanism for operation of these sources and sinks. However, personally we still believe that the phenomena of reconnection on the background of the southward component of *IMF* is the best possible candidate for that role. It should be commented that the reconnection process can happen locally, and the the multitude of sources/sinks required above can be reduced to the multitude of reconnection centers around the tail. Then, our model acquires an immunitet against dynamics of external energy environment.

So, as we mentioned above, though *CEL* model does not mandatorily require a southward *IMF* that produces reconnection on the dayside magnetosphere as an energy input, nevertheless, *CEL* does not operates without energy flow inwards the tail. Thus, the starting point of our model is the energy flow through the tail.

Then, going to the suitable physical details, the large-scale process of substorm development is characterized by the intensification of the cross-tail current and accumulation of the transferred energy in the tail lobes. The plasma sheet responds appropriately by convection closed field lines sunward and thinning under the dual effects of compression by the enhanced lobe field, and loss of flux through convection. So, the magnetosphere becomes progressively more stressed, until an expansion phase begins. Abovesaid assumes that predominant energy flow goes inwards the tail.

At y_{onset}, the change of preferable sign of the energy exchange rate to support $y = y_{out}$ takes place. It constitutes that energy predominantly begins to leave the tail, so the substorm expansion phase starts to develop. It begins with the sudden relax of the magnetic field in the near-Earth magnetotail from the stressed tail-like configuration to a more dipolar one. At the same time, the cross-tail current is being reduced. These effects can be caused by the disruption of cross-tail current sheet, energization of plasma and expansion of the plasma sheet. At this, current is diverted out of the magnetotail along the magnetic field into the ionosphere and back out into the magnetotail. The electric field generated by convection is projected along field lines onto the ionosphere driving ionospheric convection and the *DP2* ionospheric current system.

So, though presented *CEL* concept still believes that a phenomenon of reconnection plays critical role in energetics of substorm activity, however, the role of reconnection is limited and reduces to energy suppling of the tail with some amount of energy. After that the system-based processes in the tail will handle incoming energy and provide traditional structure.

3. Discussion

Last decades of research in magnetospheric substorms has not led to a consensus for the substorm process [18]. Several substorm models, mostly phenomenological, are presently under consideration. These competing models, each being justifiable on the basis of certain features of a substorm, have major differences as well as similarities among them. A synthesis substorm model is also available by extracting important components from these existing models, for example, [19].

As mentioned in [11], it is clear that the main challenge in achieving a quantitative understanding of the magnetospheric signatures of substorms is to understand the anomalous dissipation processes in the collisionless plasma of the plasma sheet. The dissipative *MHD* models have been successful in describing the large-scale features of the magnetosphere and predicting the existence of the bow shock, the magnetopause. But the *MHD* equations have failed to describe the structures of the bow shock and the magnetopause.

Novelty of this report lies in simulation which mimics development of a magnetospheric substorm based on the *CEL* self-consistent theory for energy evolution in open thermodynamic system with infinite number of conserved energy links and within this model provides evidence for the observed dynamics of a substorm.

As such, presented simulation provides general physical pattern that supports natural transformation of a chaotic energy flow through magnetosphere tail into the structured substorm reply. In this sense, we trace how the sufficiently general reasoning about configuration of energy exchange could influence to forming of a convenient structure of energy profile for a substorm.

Note that our model is a global one dealing with the tail as one indivisible system. In this aspect, *CEL* does not take any local instabilities which may contribute to energy conversion in the tail.

By so far, an increase of energy input towards the tail is unavoidable consequence of turn B_z southwards. It is the well-established fact many times confirmed in different observations. However, the question on the bidirectional character of above connection is still exists. We mean whether the changes in direction of the energy flux inwards or outwards the tail have any impact on the direction of B_z southwards or northwards. Formally, Poynting vector P is determined as the cross product, so $y \sim P \sim [\mathbf{EB}]$, where **E** is electric field vector. Then, passing on the point y_{onset}, flux P basically should change its sign either. If direction of electric field **E** does not change, then the sign for the vector **B** may also change equally. Then, the turn in B_z may follow change in direction of energy flux P and variation in P is the main reason to start energy supplying of substorm. However, a detailed discussion of this point lies beyond the scope of this paper.

It should be noted that in contrast to usual mathematical understanding of deterministic chaos, which describes the behaviour of dynamical systems highly sensitive to initial conditions, in the considered theory, we do not account initial conditions at all due to selected method of formalization. Instead, behaviour of the tail energy evolution is totally defined by internal logic of the involved system relations.

Another aspect of behaviour for chaotic systems usually associated to randomization is due to exponential growth of perturbations in the initial conditions. However again, in our case, the randomization comes inside-out as it is an attribute of employed physics.

Analyzing role of the self-organized criticality (*SOC*) in the tail note that there are solid arguments to believe that the tail plasma is in the *SOC* state indeed. In contrast that, *CEL* theory does not use the *SOC* assumption, instead it considers energy variations in the tail from the angle of full-life cycle energy evolution of the tail. One more difference is that the medium in *CEL* model does not need an assumption for presence of the charged particles, *i.e.,* approximation of plasma.

Remind that *CEL* theory states that substorm events can happen simply following internal logic of system energy development allowing to generate different magnetospheric scenarios. In this sense, *CEL* model definitely has the advantage to interpret different forms od substorm disturbances, for example, the substorms in which dramatic effects occur close to the Earth but not further down the tail [4]. Notice that based on definition of efficiency Υ as an attribute of energy conversion, we can think of variations of Υ as changes in work of the physical mechanism that generates substorms. This efficiency changes over total energy rate *y* reflecting the self-regulating system features of the tail.

In a line with above, properties of geomagnetic tail as a thermodynamic system are primary for handling of energy flows and can override other effects that con contribute to energy balance of magnetosphere.

Major conclusion of this paper is that pattern for energy development of a few substorm-like magnetosphere events can be intuitively simulated by the *CEL* model. Above results give a key for understanding of alternative high-level physical mechanism to initiate energy development of magnetospheric substorm.

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